

# Workshop on "Blow up and dispersion in Nonlinear PDEs"

Tuesday, 1 July (Opening 15:00) - Friday, 4 July, 2014

Location: WPI, Seminar Room 08.135

	Tuesday	Wednesday	Thursday	Friday
09:45 - 10:30		C. Klein	O. Ivanovici	C. Scheid
10:30 - 11:00		<i>Coffee &amp; Cake</i>	<i>Coffee &amp; Cake</i>	<i>Coffee &amp; Cake</i>
11:00 - 11:45		G. Lebeau	F. Planchon	F. Golse
15:00 - 15:45	J-C. Saut	D. Chiron	H. Luong	
15:45 - 16:00	<i>Coffee &amp; Cake</i>	<i>Coffee &amp; Cake</i>	<i>Coffee &amp; Cake</i>	<i>Coffee &amp; Cake</i>
16:00 - 16:45				

**Chiron David:** *The KP-I limit for the Nonlinear Schrödinger Equation*

In some long wave asymptotic regime, the Nonlinear Schrödinger Equation with nonzero condition at infinity can be approximated by the Kadomtsev-Petviashvili-I (KP-I) equation. We provide some justifications of this convergence for the Euler-korteweg system, which includes the Nonlinear Schrödinger Equation. In some cases, we may obtain the (mKP-I) equation. The convergence also holds for the travelling waves of the Nonlinear Schrödinger Equation when the propagation speed approaches the speed of sound. We also give some results in this direction, as well as numerical results. This talk is a survey of various results obtained with M. Maris, S. Benzoni-Gavage and C. Scheid.

**Golse François:** *The Boltzmann equation in the Euclidean space (joint work with C. Bardos, I. Gamba and C.D. Levermore)*

The Boltzmann equation is a well-known example of dissipative dynamics, because of Boltzmann's H Theorem, which is a quantitative analogue of the second principle of thermodynamics. When the Boltzmann equation is posed in the Euclidean space, the dispersion properties of the advection operator corresponding to the collisionless dynamics offsets the dissipative effect due to the collision integral. We discuss the long time behavior of the solution of the Boltzmann equation in this setting and prove the existence of a local scattering regime near global Maxwellian solutions.

**Ivanovici Oana:** *A parametrix construction for the wave equation inside a strictly convex domain*

We describe how to obtain such a parametrix by a suitable generalization of the model case which was obtained by I-Lebeau-Planchon. The procedure is however different on several points and allows for some conceptual simplifications which we will try to highlight. From this parametrix we may then get sharp dispersion estimates by degenerate stationary phase arguments. This is joint work with R. Lascar, G. Lebeau and F. Planchon.

**Klein Christian:** *Dispersive shocks in 2+1 dimensional systems*

We present a numerical study of dispersive shocks and blow-up in 1+1 and 2+1 dimensional systems from the families of Korteweg-de Vries and nonlinear Schrödinger equations.

**Lebeau Gilles:** *The fundamental solution of the wave operator on the Bethe lattice*

We compute the fundamental solution for the wave equation on the regular infinite tree with each vortex of degree 3 (the so called Bethe lattice). We get dispersive estimates and the range of values of the effective speeds of propagation. This is a joint work with Kais Ammari.

**Luong Hung:** *The focusing 3-d cubic nonlinear Schrödinger equation with potential (joint work with T. Duykearts and C. Fermanian Kammerer)*

There is a sharp condition for scattering of the radial 3-d cubic nonlinear Schrödinger equation that was given by Justin Holmer and Svetlana Roudenko. Following this spirit, we provide some similar results for this equation with potential.

**Planchon Fabrice:** *From dispersion to Strichartz: a longer journey than usual*

Usually, Strichartz estimates follow almost trivially from dispersion using duality and interpolation. For the wave equation inside a model case of a strictly convex domain, however, the resulting theorem is not sharp and we will present 2 different arguments which in some sense average over the space-time regions where swallowtail singularities (where the worse loss occur) appear and recover Strichartz estimates which would be induced by cusp-like losses. This is joint work with O. Ivanovici and G. Lebeau.

**Saut Jean-Claude:** *Weak dispersive perturbations of nonlinear hyperbolic equations*

We address the question of the influence of dispersion on the space of resolution, on the lifespan, on the possible blow-up and on the dynamics of solutions to the Cauchy problem for 'weak' dispersive perturbations of hyperbolic quasilinear equations or systems.

**Scheid Claire:** *Multiplicity of the travelling waves in the Kadomtsev-Petviashvili-I and the Gross-Pitaevskii equations*

Explicit solitary waves are known to exist for the Kadomtsev-Petviashvili-I (KP-I) equation in dimension 2 from the work of [1] and [2]. We first address numerically the question of their Morse index. The results confirm that the lump solitary wave has Morse index one and that the other explicit solutions correspond to excited states. We then turn to the 2D Gross-Pitaevskii (GP) equation which in some long wave regime converges to the (KP-I) equation. We perform numerical simulations showing that a branch of travelling waves of (GP) converges to a ground state of (KP-I), expected to be the lump. Furthermore, the other explicit solitary waves solutions to the (KP-I) equation give rise to new branches of travelling waves of (GP) corresponding to excited states. This is a joint work with D. Chiron.

[1] S. Manakov, V. Zakharov, L. Bordag and V. Matveev, Two-dimensional solitons of the Kadomtsev-Petviashvili equation and their interaction. Phys. Lett. A 63, 205-206 (1977).

[2] D. Pelinovsky and Y. Stepanyants, New multisoliton solutions of the Kadomtsev-Petviashvili equations. Pis'ma Zh. Eksp. Teor Fiz 57, no. 1 (1993), 25-29