Program
Second Wien-Graz-Bio-PDE Day

This workshop is supported by the Doctoral School “Dissipation and dispersion in nonlinear partial differential equations (nPDE)” (TU Wien and Uni Wien) funded by the Austrian Science Fund.

Location

Wolfgang-Pauli Institute at University of Vienna, UZA4, Nordbergstr. 15, 1090 Wien.

The closest subway station is “Friedensbrücke” (U4). Coming from Vienna airport, you may take the local train S7 to “Landstraße (Wien Mitte)” and change to the subway line U4 to “Friedensbrücke”. Coming from the railway station “Wien Westbahnhof”, you may take the subway line U6 to “Spittelau” and change at the station “Spittelau” to the line U4 to “Friedensbrücke”. From “Friedensbrücke”, it is just a 5 min. walk to Nordbergstr. 15.

Schedule

10:45 - 11:30	Michael Winkler (Univ. Duisburg-Essen)
11:35 - 12:05	Christof Winkler (Univ. Wien)
12:10 - 12:40	Angelika Manhart (RICAM)
12:45 - 14:00	Lunch break
14:00 - 14:45	Sebastian Novak (IST Austria)
14:50 - 15:35	Tuomo Valkonen (Univ. Cambridge)
15:35 - 16:00	Coffee break
16:00 - 16:45	Dietmar Ölz (RICAM)
16:50 - 17:20	Evangelos Latos (Univ. Graz)
17:25 - 17:55	Gilbert Raras Peralta (Univ. Graz)

Abstracts

Michael Winkler: Finite-time blow-up in the Keller-Segel system

We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \end{cases} \qquad (\star)$$

in a ball $\Omega \subset \mathbb{R}^n$ with $n \geq 2$. This system forms the core of numerous models for the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves.
The talk is mainly concerned with the phenomenon of blow-up in finite time, for which only very few examples have been detected in the literature. By providing an essentially explicit blow-up criterion it is shown that within the space of all radial functions, the set of such blow-up enforcing initial data indeed is large in an appropriate sense; in particular, this gives some rigorous evidence for the old conjecture that blow-up is a generic phenomenon in ($\star$).
One focus of the presentation is on the method through which this result is obtained. In contrast to previous approaches, it is based on a more elaborate use of the natural energy inequality associated with ($\star$), involving certain integral inequalities. In the case $n \geq 3$, for instance, an estimate of the form

$$\int_\Omega uv \leq C \cdot \left(\left\| \Delta v - v + u \right\|_{L^2(\Omega)}^{2\theta} + \left\| \frac{\nabla u}{\sqrt{u}} - \sqrt{u} \nabla v \right\|_{L^2(\Omega)} + 1 \right),$$

is shown to be valid with certain $C > 0$ and $\theta \in (0,1)$ for a wide class of smooth positive radial functions $(u,v) = (u(x), v(x))$.
Part of the results has been obtained in collaboration with Noriko Mizoguchi.

Christoph Winkler: Stochastic model explains the flatness of lamellipodia

Many cells are pushed by thin sheets, called lamellipodia, that consist of a plasma membrane wrapped around an oriented actin filament meshwork. The filaments, growing by polymerization and being capped near the front, form a quasi two-dimensional network by preferably branching parallel to the substrate. How the branching directions are chosen and thus the thinness of the lamellipodium is conserved remains unclear. We have developed a model that describes both the filament network and a fully deformable plasma membrane. In each time step the membrane shape is determined by minimizing an energy functional that takes into account the membrane's curvature, the filament ends as adhesive obstacles and biologically motivated boundary conditions. Our stochastic simulation uses the local geometric interplay between filaments and the membrane to determine the polymerization rate and the branch direction. The results show that this suffices to maintain thin lamellipodia and that no extra forces or restrictions acting on the membrane are needed.

Angelika Manhart: A finite element simulation of moving cells

Several cells use a thin, sheet-like structure called lamellipodium to crawl on surfaces. In this talk I present the numerical results of a continuous 2D model of such a structure including the various biological components such as actin filaments, adhesion complexes, myosin, cross-linkers etc. Using the finite elements method for simulation we show that the model is able to reproduce stationary and moving states of cells under various conditions. The work presented is in co-operatiion with C. Schmeiser, D. Oelz and N. Sfakianakis.

Sebastian Novak: Type-dependent diffusion and the evolution of dispersal

Typically, organisms live in a spatially extended habitat; populations disperse and interact locally with their immediate neighborhood. General mobility and directional biases of dispersal strategies determine how a population exploits spatial resources. As dispersal evolves, patterns of dispersal compete against each other and thereby adapt to the characteristics of the environment. I present a general model of type-dependent diffusion in space that contains many previous models as special cases and allows the study of different patterns of dispersal present in a population. Treating dispersal strategies as an evolutionary trait, I show that variations from a resident dispersal pattern do not involve a long-lasting advantage in the deterministic setting if the environment in homogeneous. In a finite population, however, increased mobilities are favored as a consequence of random sampling errors. In contrast, spatial heterogeneities of the habitat fuel the evolution of dispersal, causing certain dispersal strategies to be superior over others. The presented results suggest an intrinsic cost of high mobility due to an imperfect match of carrying capacity and the actual population size profile.

Tuomo Valkonen: Higher-order regularisation of diffusion tensor fields from medical MRI

Researchers in mathematical imaging have in recent years become interested in higher-order discontinuity-preserving techniques in order to overcome deficiencies in linear and first-order approaches. Namely, whereas the much studied Total Variation regularisation technique can preserve edges in images, it suffers from the stair-casing effect, essentially producing piecewise constant images. Total Generalised Variation (TGV) is presently the most promising higher-order technique that can preserve edges while also avoiding the stair-casing effect of Total Variation. It does this by optimally balancing between first- and higher-order features. In our recent work, we extended TGV to tensor fields, and studied its application to the denoising of medical diffusion tensor imaging (DTI). These arise from the combination of multiple diffusion-weighted MRI images through the Stejskal-Tanner equation, and describe the pointwise Gaussian probability distribution function for the diffusion of water molecules. By the study of DTI images, it is possible for medical practitioners to detect pathologies in the brain, for example, through deficiencies in white matter, which has

a different tensor structure from gray matter. As the MRI process is inherently noisy, it is desirable to develop good denoising approaches to help the interpretation of DTI images. This talk presents one such approach.

Dietmar Ölz: A viscous two-phase model for contractile actomyosin bundles

A mathematical model in one dimension for a non-sarcomeric actomyosin bundle featuring anti-parallel flows of anti-parallel F-Actin is introduced. The model is able to relate these flows to the effect of cross-linking and bundling proteins, to the forces due to myosin-II filaments and to external forces at the extreme tips of the bundle.

The modeling is based on a coarse graining approach starting with a microscopic model which includes the description of chemical bonds as elastic springs and the force contribution of myosin filaments. In a second step we consider the asymptotic regime where the filament lengths are small compared to the overall bundle length and restrict to the lowest order contributions. There it becomes apparent that myosin filaments generate forces which are partly compensated by drag forces due to cross-linking proteins. The remaining local contractile forces are then propagated to the tips of the bundle by the viscosity effect of bundling proteins in the filament gel.

The model is able to explain how a disordered bundle of comparatively short actin filaments interspersed with myosin filaments can effectively contract the two tips of the actomyosin bundle. It gives a quantitative description of these forces and of the anti-parallel flows of the two phases of anti-parallel F-Actin. An asymptotic version of the model with infinite viscosity can be solved explicitly and yields an upper bound to the contractile force of the bundle.

Evangelos Latos: Global dynamics of a mass conserved reaction-diffusion system.

The global dynamics of a mass conserved reaction-diffusion system are studied. First, we show the global-in-time existence of the solution with compact orbit. Next, we prove the dynamical stability of local minima associated with a variational function. This work is a collaboration with Takashi Suzuki.

Gilbert R. Peralta: Global smooth solution to a hyperbolic system arising in multiscale blood flow models

We consider a hyperbolic system of two partial differential equations in one space dimension with ODE boundary conditions describing the flow of an incompressible fluid in an elastic tube that is connected to a tank at each end. Using the local-existence theory together with entropy methods, the existence and uniqueness of a global-in-time smooth solution is established for smooth initial data sufficiently close to the constant equilibrium state. Joint work with Georg Propst.