A discrete boundedness-by-entropy method for finite-volume approximation of a certain class of cross-diffusion systems

Antoine ZUREK, Technische Universität Wien

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In this talk we will present a finite-volume scheme which allows to approximate some cross-diffusion systems. In particular, we consider cross-diffusion systems which admits a (formal) gradient flow structure given by a Boltzman-type entropy functional. This structure permits to prove at the continuous level and under suitable assumptions the existence of global-in-time bounded weak solutions. As already observed in [2, 4], the main idea for the constuction of the finite-volume scheme presented in this talk is to preserve this gradient flow structure. Then, adapting at the discrete level the so-called boundedness-by-entropy method, see [1, 3], we are able to prove the existence of bounded by below and/or above finite-volume solutions to this scheme and its convergence.
We will first explain the main ideas of the boundedness-by-entropy method and in a second time we will explain how to construct this finite-volume scheme. Then, after a presentation of our main results we will give some examples of cross-diffusion systems for which the scheme and the techniques developed in this work can be used. Finally, we will illustrate the efficiency of the scheme through numerical experiments. This work is a joint work with Ansgar Jüngel.

References

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Antoine ZUREK, Institute for Analysis and Scientific Computing, Vienna University of Technology, Wiedner Hauptstraße 8–10, 1040 Wien, Austria
antoine.zurek@tuwien.ac.at