DK Summer School 2021

September 25-30, 2021
Weissensee, Kärnten, Austria

Christian Klein

Institut de Mathématiques de Bourgogne, UBFC

Numerical approaches to nonlinear evolution equations

We present an introduction to numerical methods for nonlinear partial differential equations, mainly from hydrodynamics and nonlinear optics. The spatial part of the PDEs will be treated with spectral methods because of their excellent approximation properties for smooth functions. For periodic boundary conditions, Fourier spectral methods will be used by employing fast algorithms for the discrete Fourier transform. For more general boundary conditions, polynomial interpolants are discussed. The boundary conditions will be implemented via tau-methods. After the spatial discretization, the PDE is treated as a high dimensional system of ODEs in time, an approach referred to as method of lines. The time integration is carried out with high order finite difference schemes. We discuss the notion of stiffness of a differential system, and the stability of explicit numerical schemes. Several explicit and implicit methods adapted to stiff systems are presented. The methods are applied to concrete examples to illustrate where they are most efficient.