DK Summer School 2017

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Aspects of Applied Analysis, Functional Inequalities and Applied Probability in Many Particle Models

The subject of Kinetic Theory, the mathematical theory of many object systems, has seen great changes and growth in the past few decades. Besides its physically appealing question, Kinetic Theory has shown to be a subject that thrives on intra-disciplinary study, and connections between seemingly uncorrelated fields in Mathematics. The purpose of our mini course is to see exactly how this comes to be, especially in the setting of many particle models. During our lectures, we will focus our attention on a simplified, yet very important, model for dilute gas - the so called Kac's Model. We will explore its properties and introduce the notion of chaoticity, from which we will be able to conclude a caricature of the famous Boltzmann equation, when one considers the limit of taking the number of particles to infinity. With this at hand, we will try to follow Kac's vision and deduce a rate of convergence to equilibrium to his limit equation from the many particle model itself. Our focus will be mainly on two approaches: Spectral gap and Entropy method. While the former will be shown to be ineffective, our attention for most of the mini course will be on the latter. We will introduce links with Applied Probability, mainly via local Central Limit Theorems, to explore special states that seem natural to the problem and will be significant to the study of the entropy method. We will continue by finding links to geometric functional inequalities, with connection to the topic of optimal transportation, that give new intuition to the problem. Lastly, we will explore new notions of chaoticity and concentrations on Kac's sphere to obtain a few functional inequalities that fulfil Kac's original vision.