DK WINTER WORKSHOP 2015

Dissipation and Dispersion in Nonlinear PDEs

January 21-23 at Seehotel Rust

SCHEDLUE

Wednesday, January 21

15:00-18:00	Arrival
16:45-17:45	**DK Student meeting**
18:30	Dinner

Thursday, January 22

07:00-09:00	Breakfast
09:00-09:45	**Andreas Prohl:** *Optimal Control in Evolutionary Micromagnetism*
09:50-10:35	**Clemens Heitzinger:** *Multiscale Problems in Nanotechnology*
10:40-11:10	Coffee break
11:10-11:55	**Klemens Fellner:** *Entropy- and Duality methods for Nonlinear Dissipative PDE Models*
12:00-13:30	Lunch break
13:30-14:15	**Jan-Frederik Mennemann:** *Optimal Control of Ultracold Gases in Magnetic Microtraps*
14:20-15:05	**Sabine Hittmeir:** *Travelling waves for a nonlocal (KdV-)Burgers equation*
15:10-15:40	Coffee break
15:40-16:25	**Nicola Zamponi:** *Analytical study of degenerate cross-diffusion population models with volume filling*
16:45-17:45	**DK Faculty Meeting**
18:00	Dinner
	Departure of Guests and DK Faculty Members

Friday, January 23

Workshop "How to write a math paper"

07:00-09:00	Breakfast
09:00-10:30	Part 1 (A. Jüngel)
10:30-11:00	Coffee break
11:00-12:15	Part 2 (A. Jüngel)
12:30-13:30	Lunch break
13:30-16:00	Part 3 (B. Schörkhuber)
16:10	Departure

GENERAL INFORMATION

Location:	Seehotel Rust
	Am Seekanal 2-4, 7071 Rust
	Tel.: +43 (0)2685/3810
	Email: seehotel.rust@vivat.at
Local Organizers:	Birgit Schörkhuber, Ansgar Jüngel (TU Wien)
Contact:	birgit.schoerkhuber@tuwien.ac.at

ABSTRACTS

Klemens Fellner (University of Graz)

Entropy- and Duality methods for Nonlinear Dissipative PDE Models: Global Existence and Large Time Analysis.

Many PDE models in applied mathematics feature a dissipative structure, which can be captured, for instance, by the monotone decay (or increase) of an entropy functional. Over the last decades, exploiting such entropy structures has attracted a large amount of mathematical attention leading to such important results as the Di-Perna Lions theory of renormalised solutions to Boltzmanns equation or the Desvillettes-Villani approach of explicit convergence to equilibrium for Boltzmanns equation. Starting with a volume-surface reaction-diffusion modelling asymmetric protein localisation on stem cells, we shall discuss questions of global existence, regularity and convergence to equilibrium for systems of reaction-diffusion equations. In particular, we shall present recent results on entropy and duality methods and how they apply to the existence theory and the large time analysis of systems of reaction-diffusion equations. Moreover, we shall point out recent considerations concerning the availability of entropy functional in the case of so-called complex reaction kinetics, where no detailed balance condition holds.

Clemens Heitzinger (TU Wien)

Multiscale Problems in Nanotechnology

Many recent applications in nanotechnology give rise to multiscale problems in a natural manner. The reason is that such devices often have a fine structure, while we are interested in the properties of whole devices at a much larger length scale. An overview of recent progress in using partial differential equations to model and to simulate nanotechnological devices is given with a focus on bio- and gas sensors. The basic model equations are variants of the Poisson equation and systems of transport equations. Homogenization results are discussed as well as existence and uniqueness of the solutions. Finally numerical results are presented, ranging from parallel algorithms to comparing measurements with simulations and optimizing real-world devices.

Sabine Hittmeir (RICAM, Linz)

Travelling waves for a nonlocal (KdV-)Burgers equation

In this talk existence results for travelling wave solutions to a Korteweg-de-Vries-Burgers type of equation with a nonlocal diffusion term corresponding to a fractional differential operator of order between 1 and 2 are presented. This equation arises from formal asymptotic expansions for a shallow water flow associated to the triple-deck theory. We first demonstrate the regularising effect of the fractional differential operator by deriving existence and monotonicity properties for the travelling wave solutions to the nonlinear nonlocal conservation law in the absence of the dispersive term. In contrast, the travelling waves for the fractional KdV-Burgers equation are in general not monotone, as it is the case for the classical KdV-Burgers equation, which then requires a more complicated existence proof.

Jan-Frederik Mennemann (TU Wien)

Optimal Control of Ultracold Gases in Magnetic Microtraps

A Bose-Einstein condensate is a state of matter formed by a dilute gas of bosons cooled to extremely low temperatures. The mean-field dynamics of a coherent Bose-Einstein condensate is well described by the time-dependent Gross-Pitaevskii equation. We consider Bose-Einstein condensates in magnetic confinement potentials realized on atom chips. The confinement potentials are parameterized by a single or several control parameters lambda. Our aim is to find an optimal time evolution of lambda which steers the condensate from an initial state at time zero to a desired state at final time T. Due to the high computational cost, this kind of quantum control problem has so far been considered in one spatial dimension only. In this talk I explain our approach to solve the full three-dimensional problem. The algorithm is applied to various situations, where 1D simulations can only be applied approximately or not at all. Moreover, it is shown that the three-dimensional model reveals subtle physical effects which can not be explained by simple one-dimensional calculations.

Andreas Prohl (Universität Tübingen)

Optimal Control in Evolutionary Micromagnetism

We consider an optimal control problem subject to the LandauLifshitzGilbert equation (LLG)

$$\mathbf{m}_t = -\alpha\, \mathbf{m} \times (\mathbf{m} \times \Delta \mathbf{m}) + \mathbf{m} \times (\Delta \mathbf{m} + \mathbf{u})$$

which describes the evolution of magnetizations $\mathbf{m}$ in $\mathbb{S}^2$. Here $\mathbf{u} : [0,T] \times \Omega \to \mathbb{R}^3$ is an applied field which is optimized according to some quadratic functional. The problem is motivated in order to control switching processes in ferromagnets. I start with a survey of existing numerical schemes which approximate solutions of LLG. A main focus here is to properly discretize the sphere property of solutions $\mathbf{m}$. Then, I discuss the optimality system for the optimal control problem, and a semi-discretization of it. I discuss convergence of the latter method. Computational studies will be shown. This is joint work with T. Dunst, M. Klein (U Tübingen) and A. Schäfer (TU Chemnitz).

Nicola Zamponi (TU Wien)

Analytical study of degenerate cross-diffusion population models with volume filling.

We present a broad class of population models with nonlinear degenerate cross-diffusion structure and volume-filling. The existence of bounded non-negative weak solutions to the system is shown by semi-discretizing the system in time and by adding a suitable regularizing term. A discrete entropy inequality allows to obtain gradient estimates which are uniform with respect to the time step and the regularization parameter. In order to overcome the difficulties arising from the degeneracy and the nonlinearity, a known compactness result is generalized, allowing to prove strong convergence results for the variables of interest. The positivity of the solvent density is obtained under additional assumptions. The convergence of the solution to the steady state as $t \to \infty$ is proved by means of suitable convex Sobolev inequalities and an ad-hoc application of Egorovs theorem for measurable functions on finite-measure sets. The uniqueness of bounded weak solutions for the system in a particular case is shown by applying the H^{-1} method and by proving a suitable inequality for the relative entropy between two solutions, which is a consequence of the convexity property of the Fisher information.