DK WORKSHOP

Recent Contributions of Women to PDEs

November 28-30, University of Vienna, Austria

SCHEDULE

Monday, November 28

09:30-10:15	Anne Nouri	*On the Boltzmann-Nordheim boson equation*
10:15-11:00	Helen Byrne	*PDEs in Mathematical Biology: Transforming Something Old Into Something New?*
11:00-11:30	Coffe break	
11:30-12:15	Marie Doumic-Jauffret	*Asymptotic behaviour of critical cases in the fragmentation and growth-fragmentation equation - cyclic and non steady behaviours*
12:15-14:00	Lunch break	
14:00-14:45	Angela Stevens	*TBA*
14:45-15:30	Marie-Therese Wolfram	*Cross-diffusion systems with excluded volume effects and asymptotic gradient flows*
16:00-18:00	Postersession	

Tuesday, November 29

09:30-10:15	Barbara Niethammer	*Instabilities and oscillations in coagulation equations*
10:15-11:00	Mechthild Thalhammer	*Commutator-free Magnus integrators combined with operator splitting methods and their areas of application*
11:00-11:30	Coffe break	
11:30-12:15	Carola Schönlieb	*Nonlinear PDEs for customised image analysis*
12:15-14:00	Lunch break	
14:00-14:45	Ilaria Perugia	*Space-time Trefftz approximation of wave equations*
	Dinner	

Wednesday, November 30

09:30-10:15	Irene Fonseca	*Second Order Gamma-Convergence for the Modica Mortola Functional*
10:15-11:00	Ariane Trescases	*Cross-diffusion and competitive interaction in Population dynamics*
11:00-11:30	coffe break	
11:30-12:15	Sabine Hittmeir	*Multiscale asymptotics and analysis of moist atmospheric flows*
12:15-13:00	Maria Giovanna Mora	*The equilibrium measure for a nonlocal dislocation energy*

GENERAL INFORMATION

Location:	Faculty of Mathematics, University of Vienna Oskar-Morgenstern-Platz 1, 1090 Wien
Local Organizers:	Stefan Schuchnigg, Christian Schmeiser, Ansgar Jüngel
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ABSTRACTS Monday

Anne Nouri (Institut de Mathématiques de Marseille)

On the Boltzmann-Nordheim boson equation

The Cauchy problem is considered for the Boltzmann-Nordheim equation in a slab with two-dimensional velocity space and pseudo-Maxwellian forces. Strong solutions are obtained for large initial data in an L^1 intersected with L^∞ setting. The main results are existence, uniqueness and stability of solutions conserving mass, momentum and energy that explode in L^∞ if they are only local in time.

Helen Byrne (University of Oxford)

PDEs in Mathematical Biology: Transforming Something Old Into Something New?

The application of mathematics to biology has long served as a rich source of non-standard PDE models. These range from moving boundary problems describing the growth of avascular tumours to mechano-chemical models of pattern formation. As the field of mathematical biology matures, it is natural to question the theoretical and biological assumptions on which such models are based. In this talk, I will illustrate these points by presenting results from recent work. First, I will focus on angiogenesis, the process by which new blood vessels grow from existing ones; I will show how existing (and new) PDE models of angiogenesis can be derived by coarse-graining discrete, cell-based models. I will then introduce new PDE models for tissue growth that were developed to investigate how mechanical stimuli can regulate cell proliferation and that have been used to study wound healing, tumour growth and tissue engineering.

Marie Doumic-Jauffret (INRIA)

Asymptotic behaviour of critical cases in the fragmentation and growth-fragmentation equation - cyclic and non steady behaviours

The long-time asymptotics of the fragmentation and growth-fragmentation equations have been studied by many authors, mainly proving convergence toward a steady behaviour (possibly with exponential growth) under balance assumptions on the coefficients. Exponential speed of convergence has also been established under more restrictive assumptions. We focus here on two limit cases where no such behaviour occurs. The first case is such that the balance assumptions between growth and division are not satisfied. We show that a specific dynamics emerge, where the initial condition continues to play a major role in the asymptotic profile. In the second case, emblematic of bacterial division cycle, a lack of dissipativity in the operator leads to a convergence toward a periodic behaviour.

Angela Stevens (University of Münster)
TBA

TBA

Marie-Therese Wolfram (University of Warwick)
Cross-diffusion systems with excluded volume effects and asymptotic gradient flows

In this talk we discuss the analysis of a cross-diffusion PDE system for a mixture of hard spheres, which was derived by Bruna and Chapman from a stochastic system of N interacting Brownian particles using the method of matched asympototics. Gradient-flow techniques have become a well established tool to study these kind of nonlinear PDEs. Hence expressing a nonlinear diffusion equation as a gradient flow of an entropy is a very desirable feature. The PDE system under consideration satisfies a gradient flow structure if particles have the same size. For particles of different size we can interpret the equations as an asymptotic gradient flow structure (which results from the asymptotic expansion in the derivation). We shall use this asymptotic gradient flow structure to provide existence of stationary solutions and stability close to equilibrium. Furthermore we discuss global in time existence for the full gradient flow system and illustrate the behavior of the model with various numerical simulations.

ABSTRACTS Tuesday

Barbara Niethammer (University of Bonn)

Instabilities and oscillations in coagulation equations

Smoluchowski's classical coagulation equation can describe mass aggregation phenomena in a large variety of applications, such as aerosol physics, polymerization, population dynamics, or astrophysics. The model consists of a nonlinear integral equation for the number density of clusters and involves a rate kernel that accounts for the microscopic details of the specific aggregation process. Of particular relevance is to understand whether the long-time behaviour of solutions is universal. In this talk I will in particular discuss the case of diagonally dominant kernels of homogeneity one. Here formal arguments lead to the conjecture that for large times the coagulation equation can be seen as a regularization of the Burgers equation. In contrast to diffusive regularization, however, we obtain phenomena such as instability of the constant solution or oscillatory traveling waves.

Mechthild Thalhammer (University of Innsbruck)

Commutator-free Magnus integrators combined with operator splitting methods and their areas of application

In this talk, I shall introduce the class of commutator-free Magnus integrators for non-autonomous linear evolution equations and identify different areas of application. Commutator-free Magnus integrators are (formally) given by a composition of several exponentials that comprise certain linear combinations of the values of the defining operator at specified nodes. Avoiding the costly evaluation of commutators, they provide a favourable alternative to standard Magnus integrators, in particular for large-scale applications.

Non-autonomous linear evolution equations also arise as a part of more complex problems, for instance in connection with nonlinear evolution equations of the form $u'(t) = A(t)u(t) + B(u(t))$. A natural approach is thus to apply operator splitting methods combined with commutator-free Magnus integrators. Relevant applications include Schrdinger equations with space-time-dependent potential describing Bose-Einstein condensation or diffusion-reaction systems with additional multiplicative noise modelling pattern formation.

Carola Schönlieb (University of Cambridge)

Nonlinear PDEs for customised image analysis

TBA

Ilaria Perugia (University of Vienna)

Space-time Trefftz approximation of wave equations

A space-time finite element method for linear wave propagation problems will be presented. The special feature of the scheme is that it is a Trefftz method, namely that trial and test functions are solution of the partial differential equation to be discretised in each element of the (space-time) mesh. The method is defined on unstructured meshes whose internal faces do not need to be aligned to the space-time axes. The Trefftz approach can be used to improve and ease the implementation of explicit schemes based on "tent-pitched" meshes. A priori error bounds for general Trefftz spaces are proved, and concrete discretisations based on piecewise polynomials that satisfy the wave equation elementwise are presented. This is a joint work with Andrea Moiola (University of Reading).

ABSTRACTS Wednesday

Irene Fonseca (Carnegie Mellon University, Pittsburgh)
Second Order Gamma-Convergence for the Modica Mortola Functional

The asymptotic behavior of an anisotropic Cahn-Hilliard functional with prescribed mass and Dirichlet boundary condition is studied when the parameter that determines the width of the transition layers tends to zero. The first order term in the asymptotic development by Gamma-convergence is well-known, and is related to a suitable anisotropic perimeter of the interface. Here it is shown that, depending on symmetry and growth hypotheses on the double well potential, the second order term in the Gamma-convergence expansion is zero. Nonlocal higher order singular perturbations are also considered. Slow motion is addressed, and related estimates of the rate of convergence of solutions of the associated Allen-Cahn equation to the minimum value are discussed.

Ariane Trescases (University of Cambridge)
Cross-diffusion and competitive interaction in Population dynamics

In Population dynamics, reaction-cross diffusion systems model the evolution of populations of competing species with a repulsive effect between individuals. For these strongly coupled, often nonlinear systems, a question as basic as the existence of solutions appears to be extremely complex. We introduce an approach based on the most recent extensions of duality lemmas and on entropy methods. We prove the existence of weak solutions in a general setting of reaction-cross diffusion systems, as well as some qualitative properties of the solutions.

Sabine Hittmeir (University of Vienna)
Multiscale asymptotics and analysis of moist atmospheric flows

Model reductions in meteorology by scale analysis are inevitable and therefore have a long history in meteorology. The key technique for a systematic study of complex processes involving the interaction of phenomena on different length and time scales is multiple scales asymptotics. Of particular interest here are hot towers, which are large cumulonimbus clouds that live on small horizontal scales having a diameter of the order of one kilometer. It is common belief by now that these hot towers are to a great extent responsible for the vertical heat transport into the upper troposphere within the innertropical convergence zone. Due to their major contribution to the energy transport it is extremely important to develop a good understanding of their life cycles. Moreover these deep convective clouds constitute the building blocks of intermediate scale convective storms, which we study in a next step by incorporating the setting of organised convection into the multiscale approach. This requires not only the introduction of coordinates allowing for an individual tilt for each columnar

cloud, but also new systematic averaging procedures, which enable us to quantify the modulation of the larger scale flow by the moisture processes in the small scale regions.
While the just described multiscale asymptotics are purely formal we also proceed further in the rigorous analysis of the atmospheric flow models with moisture and phase transitions. We study the global existence and uniqueness of solutions for the moisture balance equations coupled to the thermodynamic equation building the basis for the above expansions, where in a first step we assume the flow field to be given.

Maria Giovanna Mora (University of Pavia)
The equilibrium measure for a nonlocal dislocation energy

In this talk I will discuss the minimum problem for a nonlocal energy, that describes the interaction of a system of positive edge dislocations in the plane, in the many-particle limit. The interaction potential is given by the sum of a logarithmic potential and of an anisotropic term, that makes the potential non-radial. The purely logarithmic potential has been studied in a variety of contexts (Ginzburg-Landau vortices, Coulomb gases, random matrices, Fekete sets) and it is well known that, in this case, the equilibrium measure is given by the so-called circle law. I will show that the presence of the anisotropic term in the potential changes dramatically the nature of the equilibrium measure, which turns out to be supported on a one-dimensional set. This will be proved by an explicit characterization of the equilibrium measure.