Reaction-Diffusion Equations with Spatially Distributed Hysteresis

Mark Curran, Freie Universität Berlin

January 27, 2016

In this talk, we will treat the equation

$$u_t = \Delta u + f(u, \mathcal{H}(u)), \tag{1}$$

on a bounded domain in $\mathbb{R}^n$, where $n \geq 2$. Here, u represents a diffusing substance and $\mathcal{H}(u)$ is a hysteresis operator defined at every spatial point. Such equations model processes where the non-diffusing substance $\mathcal{H}(u)$ can be in one of two states, and the switching mechanism between states is determined by a hysteresis law. These equations model a variety of biological and chemical processes that exhibit spatial-temporal patterns.

Numerical simulations of such models are in agreement with experiment, however questions of the existence and uniqueness of solutions, as well a rigorous explanation of the mechanisms for pattern formation remain open.

The set of points where $\mathcal{H}(u)$ is in one state or the other naturally segregates the domain into two subdomains. Moreover, the switching mechanism implies that these subdomains are separated by free boundaries.

Recently, the well-posedness of 1 for the case $n = 1$ was established for so called *transverse* initial data. In this talk I will present conditions on the initial data and the free boundary that guarantee the existence and uniqueness of solutions for $n \geq 2$. I will also describe the regularity of solutions and their connection to slow-fast systems.