DK Seminar

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TU Wien, Freihaus, green area, 4th floor, SEM 101C

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Optimal additive Schwarz preconditioning for the hp-BEM: The hypersingular integral operator in 3D

For many linear elliptic boundary value problems the boundary element method (BEM) provides an efficient way to discretize and solve it. The hypersingular integral operator appears when trying to solve problems with Neumann or mixed boundary conditions. Just as in the finite element method (FEM) the accuracy of approximation can be increased by reducing the mesh size h (h-refinement), increasing the polynomial degree p of the ansatz space (p-refinement) or a combination of the two (hp-refinement). The discretization leads to large linear systems of equations $Ax = f$, for which iterative solvers may be used. For SPD matrices A the performance of those solvers depends on the condition number $\kappa := \frac{\lambda_{max}}{\lambda_{\min}}$ of A. In order to improve convergence speed, the strategy of preconditioning is to replace the matrix A with the matrix $B^{-1}A$ which has a much lower condition number, and solve $B^{-1}Ax = B^{-1}f$ instead.
In this talk I propose a preconditioner for the hypersingular integral operator, based on an overlapping additive Schwarz approach. The abstract Schwarz theory for this family of preconditioners allows for an elegant analysis based on space decompositions. The proposed preconditioner is based on a space decomposition into piecewise linears and spaces of high order polynomials supported by the vertex patches. This decomposition results in uniformly bounded (w.r.t h and p) condition numbers for the preconditioned system. The space of piecewise linears cab be further decomposed in a multilevel fashion, even for locally refined meshes without compromising the robustness of the resulting preconditioner.