DK Seminar

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University of Vienna, Faculty of Mathematics, OMP 1, HS 2

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Microstructures: analysis and numerics of periodic solutions

Microstructures are usually defined as structures that can be observed between the macroscopic and the atomic scale. Their occurrence in special classes of materials (such as shape memory alloys) is the key to understand the complicated and surprising properties of these materials. In the literature, multi-scale microstructures are mathematically described by variational models. Their occurrence is then conjectured as an attempt to minimize a certain energy functional. However, this model is quite hard to analyze by variational techniques even in simple settings. We propose to use a different approach based on geometric singular perturbation theory with the goal to obtain further insight, e.g. on critical points (not just the minimizers) of the functional. The Euler-Lagrange equation associated with the functional can be written as a 4-dimensional singularly perturbed Hamiltonian system of ODEs. In my previous talk, I had illustrated the initial slow-fast analysis of this system. In this talk, I will show the results we obtained since then, starting from the reduction of the system to a 3-dimensional one by restricting it to the hyper-surface corresponding to a fixed value of the Hamiltonian. I will explain the construction of the singular periodic orbit ($\varepsilon = 0$) for different values of the Hamiltonian. Finally, I will show that these singular periodic orbits persist for $0 < \varepsilon \ll 1$, giving rise to a family of periodic orbits. These results will be illustrated and confirmed by numerical computations based on AUTO (a software package for continuation and bifurcation problems in ODEs).