DK Seminar

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University of Vienna, Faculty of Mathematics, OMP 1, HS 2

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Approximation-schemes for Schrödinger-type equations including turning points

To resolve highly oscillating solutions of a differential equation, a very fine grid for the step size is needed. This results in high computational costs and is insufficient if a large number of solutions need to be calculated simultaneously. In my talk I will cover a numerical method to efficiently and accurately approximate a solution of the stationary 1-dimensional Schrödinger equation $\varepsilon^2 \varphi_{xx}(x) + a(x)\varphi(x) = 0$ as presented in [1]. There the approach is to use a second order WKB-type transformation and filter out the dominant part of the oscillations. The method used is in many applications even asymptotically correct w.r.t. the small parameter ε. However, it is only done for a coefficient function $a(x)$ that is positive and bounded away from zero, as this approach breaks down at so called turning points x_0 where $a(x_0) = 0$. For the problem at hand including turning points, we try an approach motivated by Langer [2]. The asymptotic Ansatz leads to the use of a modification of Bessel functions of the third kind, the Hankel functions.

References

[1] Anton Arnold ; Naoufel Ben Abdallah ; Claudia Negulescu: *WKB-Based Schemes for the Oscillatory 1D Schrödinger Equation in the Semiclassical Limit*, SIAM Journal on Numerical Analysis, 2011.

[2] Rudolph E. Langer: *On the asymptotic solutions of ordinary differential equations, with an application to the Bessel functions of large order*, Transactions of the American Mathematical Society, 1931.