Global existence analysis for degenerate energy-transport models for semiconductors

A class of energy-transport equations without electric field under mixed Dirichlet-Neumann boundary conditions is analyzed. The system of degenerate and strongly coupled parabolic equations for the particle density and temperature arises in semiconductor device theory.
The global-in-time existence of weak nonnegative solutions is shown. The proof consists of a variable transformation and a semi-discretization in time such that the discretized system becomes elliptic and semilinear. Positive approximate solutions are obtained by Stampacchia truncation arguments and suitable cut-off test functions.
Nonlogarithmic entropy inequalities yield gradient estimates which allow for the limit of vanishing time step sizes. Exploiting the entropy inequality, the long-time convergence of the weak solutions to the constant steady state is proved. Because of the lack of appropriate convex Sobolev inequalities to estimate the entropy dissipation, only an algebraic decay rate is obtained. Numerical experiments indicate that the decay rate is typically exponential.