ANALYSIS OF A CROSS-DIFFUSION HERDING MODEL

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Our aim is to analyse and to understand the following cross-diffusion model

$$\begin{cases} \partial_t u = div(\nabla u - g(u)\nabla v) \\ \partial_t v = div(\delta \nabla u + \kappa \nabla v) + f(u) - \alpha v \end{cases}$$

with no-flux boundary conditions and initial conditions in a bounded domain $\Omega \in \mathbb{R}^d$, $\alpha > 0$. If $\delta = 0$, $g(u) = u$, $f(u) = u$ this model describes the chemotactic evolution of a cell density u depending on the chemical substance v. If $\delta = 0$, $g(u) = u(1-u)$, $f(u) = u$ or $f(u) = u(1-u)$ the system models the crowd motion and herding. We may interpret u as an information density, influenced by the potential v which expresses the tendency of people to adapt their individual opinion to the majority opinion. The term $g(u) = u(1-u)$ forces the information density to attain value in the interval $[0, 1]$. If $\delta > 0$ and the functions f and g are chosen in an appropriate way, the above system possesses an entropy functional and we can prove the global existence of weak solutions.

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