

Long-Time Asymptotics for the Korteweg-de Vries Equation via Steepest Descent

Abstract

I describe the Korteweg-de Vries (KDV) equation $q_t = -q_{xxx} + 6qq_x$, and its longtime behavior, to give an introduction into a method - the Inverse Scattering Transform (IST) and Nonlinear Steepest Descent - which is applicable for a broad class of functions. At first i will show the dispersive behavior of the linear part of the equation with help of Fourier transform and the method of linear steepest descent. Secondly i will show the behavior of the nonlinear part of the KdV. Over time nonlinear and dispersive parts become balanced and form one soliton solutions. I make a short sketch of the IST and give the reason, why the IST is possible, with help of the Lax Pair. The difficult step of IST is the recovery of the wave function from some scattering data, which is an Riemann-Hilbert problem with jump about the real axis. We can solve it with help of nonlinear steepest descent similar to the linear method.