Computation of water waves with constant vorticity

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Two-dimensional, finite-depth periodic water waves with constant vorticity and large amplitude are computed, based on the work of Constantin and Strauss, see [1]. This involves the classical water wave problem described by the Euler equations with a free surface, under the influence of gravity, over a flat bottom. The analysis is based on a nonlinear domain transformation, i.e. the Dubreil-Jacotin transformation, which transforms the free boundary problem into a fixed boundary problem and the Bernoulli's law into a nonlinear boundary condition; the constant that appears in this boundary condition is now considered as a bifurcation parameter.
Considering the numerical implementation, we formulate an optimization problem, in order to derive solutions of the latter boundary value problem that correspond to waves of large amplitude. Both qualitative and quantitative results, which characterize the waves, are presented, in terms of some physical parameters, such as the vorticity, the relative mass flux and the total mechanical energy.

[1] A. Constantin and W. Strauss, Exact Steady Periodic Water Waves with Vorticity, Communication on Pure and Applied Mathematics, LVII, 481-527, 2004