Efficient additive Schwarz preconditioning of the hypersingular integral equation on locally refined triangulations

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We consider the hypersingular integral equation for the 2D and 3D Laplacian. It is well-known that the condition number of the Galerkin matrix grows as the mesh is refined. The situation is even worse on locally refined meshes, where the condition number grows with the number of elements as well as the global mesh-size quotient $h_{\max}/h_{\min}$. Therefore, the developement of efficient preconditioners is a necessary and important task.

In this talk, we present the results of our recent work [1], where we consider a (local) multilevel diagonal preconditioner. The basic idea of this preconditioner is to consider only newly created nodes in $\mathcal{T}_{\ell+1}\backslash\mathcal{T}_\ell$ plus their immediate neighbours for preconditioning. For uniform refinement, it was proved in [3] that multilevel diagonal preconditioners are efficient in the sense, that the condition number of the preconditioned system is independent of the number of levels and the mesh-size. On locally refined triangulations such a result was unknown.

Basically, the proof consists of providing a stable subspace decomposition for the fractional order Sobolev space $H^{1/2}$ by means of a variant of the Scott-Zhang projection [2]. In the frame of 2D-FEM, a stable subspace decomposition of H^1 has been considered in [4], and we transfer and extend these ideas to $H^{1/2}$.

We show efficiency of the (local) multilevel diagonal preconditioner in the sense that the condition number of the resulting system is independent of the mesh-size and the number of levels. Numerical examples on closed and open boundaries underline our theoretical results.

References

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