Causality analysis for frequency-dependent attenuation of acoustic waves

Causality is a very important concept in many areas, including physics, engineering, digital signal processing and it has practical meaning. To correctly model an attenuated wave, we need to ensure causality of the wave. Causality imposes relations between attenuation and dispersion, which are known as the Kramers-Kronig (K-K) relations between the real and imaginary parts of the complex wave number $\kappa(\omega)$. However, the K-K relations are necessary but not sufficient for causality of a wave. Kowar *et.al.* have given a necessary and sufficient condition for causality, but it requires the continuation of $\kappa(\omega)$ to the upper half-plane, and is hard to check.

In this talk, I demonstrate our result about causality for frequency-dependent attenuation of acoustic waves. I will first review the definitions and known results, and then give a new necessary and sufficient condition which involves only the value of $\kappa(\omega)$ on the real axis, so is easy to check. Then in the last part of the talk I will give some examples to demonstrate the simplicity of the new characterization.