Integration of Vlasov-type equations

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In astro- and plasma physics the behavior of a collisionless plasma is modeled by the Vlasov equation

$$\partial_t f(t, \boldsymbol{x}, \boldsymbol{v}) + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f(t, \boldsymbol{x}, \boldsymbol{v}) + \boldsymbol{F} \cdot \nabla_{\boldsymbol{v}} f(t, \boldsymbol{x}, \boldsymbol{v}) = 0,$$

a kinetic model that is posed in a 3+3 dimensional phase space. For its numerical solution, we combine a (grid based) discontinuous Galerkin approximation in space with an operator splitting scheme in time.

A rigorous convergence analysis can be conducted, for example, for the 1+1 dimensional Vlasov–Poisson equations. It is shown that, for sufficiently regular initial data, the error of the numerical approximation is of order

$$\mathcal{O}\left(\tau^2 + h^{\ell+1}/\tau + h^{\ell+1}\right),$$

where τ is the size of a time step, h is the cell size, and ℓ the order of the discontinuous Galerkin approximation.

It is well known that piecewise constant approximations in velocity space lead to a recurrence phenomenon that is purely numerical in origin. We will present a number of numerical simulations which show that a recurrence-like effect, originating from the finite cell size, is still visible even for higher order approximations.

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