

On the inverse scattering transform method for the Korteweg–De Vries equation with steplike initial data

Abstract

We study the Korteweg-de Vries (KdV) equation $q_t = -q_{xxx} + 6qq_x$, with steplike initial data of the form $q(x) \rightarrow c_+$ as x goes to infinity and $q(x) \rightarrow c_-$ as x goes to minus infinity, with first moment finite, a.e. $\int_0^\infty (1 + |x|)(|q(x) - c_+| + |q(-x) - c_-|)dx < \infty$. Therefore we use the Inverse Scattering Transform (IST) and consider the direct/inverse scattering problems for one dimensional Schrödinger equation with steplike potential. The main aim of the talk is to prove the uniqueness of the solution. Therefore we need some decaying behavior, which follows, if the reflection coefficients of the spectral problems are continuous. Using the idea of Guseinov, we can show this by proving some technical property of the Wronskian, which is the main theorem of this talk. This together with other qualities we get the necessary and sufficient conditions of the scattering data, which allows us to obtain an unique solution of the problem.