MULTISCALE PATTERNS FOR A SINGULARLY PERTURBED VARIATIONAL PROBLEM

In the last few decades the phenomenon of so called “microstructures”, i.e. structures between the macroscopic and the atomic scale, has been studied intensively. This interest is due to the fact that microstructures influence in a crucial way the macroscopic behavior of materials in a wide range of applications. One of these, as we will briefly show, is concerned with solid-solid phase transitions in shape memory alloys. Microstructures are modeled and analyzed mathematically as the response of such systems in order to minimize their energy. This usually leads to variational problems with a non-convex functional, which causes non-uniqueness of minimizers. To obtain better agreement with the actual states observed in experiments, various types of regularizations are applied to the functional. Typically, these regularizations act on smaller scales, leading to singularly perturbed variational problems.

We study a particular one-dimensional problem of this type, which is known to have solutions of a complicated multi-scale structure. Complementing the existing results based on variational methods, we use a different approach for the study of this problem, starting from the Euler-Lagrange equation associated to the functional. This fourth order equation is rewritten as a singularly perturbed first order system, which is analyzed by methods from geometric singular perturbation theory. The aim of our work is to describe and understand these patterns from the dynamical systems and geometrical point of view.