Hardy Space Infinite Elements for Time-Harmonic Wave Equations with Phase Velocities of Different Signs

Abstract We consider time harmonic wave equations in cylindrical waveguides with physical solutions for which the signs of group and phase velocities differ. Standard transparent boundary conditions, e.g. the Perfectly Matched Layers (PML) method select modes with positive phase velocity, and hence they yield stable, but unphysical solutions for such problems.

We derive an infinite element method for a physically correct discretization of such wave-guide problems which is based on a Laplace transform in propagation direction. In the Laplace domain the space of transformed solutions can be separated into a sum of a space of incoming and a space of outgoing functions where both function spaces are curved Hardy spaces. The curved Hardy space is constructed such that it contains a simple and convenient Riesz basis with moderate condition numbers.

In this talk the new method is discussed for a one-dimensional fourth order model problem. It does not use a modal separation and works on an interval of frequencies. Numerical analysis and experiments exhibit exponential convergence and moderate condition numbers.