Status Seminar
Dissipation and dispersion in nonlinear PDEs

OPTIMAL CONVERGENCE OF AN ADAPTIVE ALGORITHM FOR THE POISSON EQUATION

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Abstract. Given a partial differential equation and a numerical algorithm to solve it, the overall goal is to compute a satisfying approximation of the unknown exact solution while investing a minimal amount of computational power and time. In this spirit, we present a standard adaptive algorithm of the form

$$\boxed{\texttt{solve}} \longrightarrow \boxed{\texttt{estimate}} \longrightarrow \boxed{\texttt{mark}} \longrightarrow \boxed{\texttt{refine}}$$

and discuss its application to the toy model problem

$$\begin{aligned} -\Delta u &= f \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^d$ is a bounded domain. The adaptive algorithm generates a sequence of discrete approximations $U_\ell \approx u$. We show that important features like convergence, i.e.

$$\lim_{\ell \to \infty} U_\ell = u$$

or convergence with algebraic rates, i.e.

$$\|u - U_\ell\| \lesssim (\text{number of unknowns in step } \ell)^{-s}$$

hold for the adaptive algorithm. Moreover, the convergence rate s is determined only by the data given in (1), and optimal for the problem at hand.