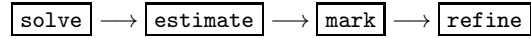


**OPTIMAL CONVERGENCE OF AN ADAPTIVE ALGORITHM FOR  
THE POISSON EQUATION**

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ABSTRACT. Given a partial differential equation and a numerical algorithm to solve it, the overall goal is to compute a satisfying approximation of the unknown exact solution while investing a minimal amount of computational power and time. In this spirit, we present a standard adaptive algorithm of the form



and discuss its application to the toy model problem

$$(1) \quad \begin{aligned} -\Delta u &= f && \text{in } \Omega, \\ u &= 0 && \text{on } \partial\Omega, \end{aligned}$$

where  $\Omega \subseteq \mathbb{R}^d$  is a bounded domain. The adaptive algorithm generates a sequence of discrete approximations  $U_\ell \approx u$ . We show that important features like convergence, i.e.

$$\lim_{\ell \rightarrow \infty} U_\ell = u$$

or convergence with algebraic rates, i.e.

$$\|u - U_\ell\| \lesssim (\text{number of unknowns in step } \ell)^{-s}$$

hold for the adaptive algorithm. Moreover, the convergence rate  $s$  is determined only by the data given in (1), and optimal for the problem at hand.