An extended midpoint scheme for the Landau-Lifshitz-Gilbert equation in computational micromagnetics

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Micromagnetic phenomena on a ferromagnetic sample $\Omega \subset \mathbb{R}^3$ are described by the time-dependent Landau-Lifshitz-Gilbert equation (LLG). Besides a geometric nonlinearity, the reliable and effective numeric integration faces the following problems: first, the continuous solution $\boldsymbol{m} : \Omega \to \mathbb{R}^3$ satisfies a non convex side-constraint $|\boldsymbol{m}| = 1$; second, each time-step requires the computation of the so-called stray field and thus leads to a coupling with an elliptic PDE in full space. The discretization in space employs lowest-order Courant finite elements, where the side-constraint is satisfied in all vertices of the underlying discretization. The approximate stray field is computed via a FEM-BEM approach to cope with the unbounded domain. In our talk, we discuss the extension of the midpoint scheme proposed in [1], where only the so-called exchange contribution (Laplace operator) is treated implicitly, while the lower-order terms (including the computationally expensive stray field) are treated explicitly in time. The resulting scheme is still unconditionally convergent to weak solutions of LLG and formally preserves second-order accuracy in time. Numerical experiments rely on an appropriate coupling of the Netgen/NGSolve package [2] with the BEM++ library [3].

References

[1] Bartels, S., Prohl, A., 2008. Convergence of an implicit finite element method for the Landau-Lifshitz-Gilbert equation. SIAM J. Numer. Anal. 44 (4), 1405–1419 (electronic)

[2] Schöberl, J. NGSolve Finite Element Library. https://sourceforge.net/projects/ngsolve/

[3] Śmigaj, W., Betcke, T., Arridge, S., Phillips, J., Schweiger, M., 2015. Solving Boundary Integral Problems with BEM++. ACM Trans. Math. Software 41 (2), Art. 6, 40 pp.