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DK Seminar

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University of Vienna

Oskar-Morgenstern-Platz 1, WPI, 8th floor, Seminarroom.

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Rate optimality of adaptive algorithms

This work aims first at the development of an axiomatic framework for the proof of optimal convergence rates for adaptive algorithms, with the main field of application being the finite element method and the boundary element method. Second, the axiomatic view allows refinements of particular questions like the avoidance of (discrete) lower bounds, inexact solvers, inhomogeneous boundary data, or the use of equivalent error estimators. Three axioms which are related to the estimator guarantee optimal convergence rates in terms of the error estimator for any refinement strategy which satisfies additional three triangulation related axioms. Compared to the state of the art in the literature (except for the recent own work [Feischl et al.,2014] which is partially generalized), the improvements of this work can be summarized as follows:

First, a general and completely abstract framework is presented which covers the existing literature on rate optimality of adaptive algorithms. The abstract analysis covers linear as well as nonlinear problems and is independent of the underlying (conforming, non-conforming, or mixed) finite element or boundary element method. Solely, the error estimator, considered as a function of the underlying triangulation, is used and analyzed. This allows to formulate axioms which are not restricted to any concrete model assumption.

Second, efficiency of the error estimator is neither needed to prove convergence nor quasi-optimal convergence behavior of the error estimator. In this work, efficiency exclusively characterizes the approximation classes involved in terms of the best-approximation error and data resolution. Therefore, the upper bound on the optimal marking parameters does not depend on the efficiency constant.

Third, some general quasi-Galerkin orthogonality is not only sufficient, but also necessary for the R-linear convergence of the error estimator, which turns out to be necessary itself when it comes to optimal complexity estimates. The latter means the optimality of the adaptive algorithm when considering the overall cost of the algorithm (which includes the computation of all previous steps) and is proved as a by-product of rate optimality.

Fourth, we circumvent the use of the overlay estimate of the refinement strategy, which is a standard assumption in the recent literature, to include popular refinement schemes like red-green-blue refinement into the analysis.

Finally, the general analysis allows for equivalent error estimators and inexact solvers as well as different non-homogeneous and mixed boundary conditions and is even employed to prove convergence of some novel adaptive geometry approximation for a certain boundary element method.