Seminar

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Wolfgang Pauli Institute, University of Vienna,
Oskar-Morgenstern-Platz 1, SR 08.135

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Analysis of travelling waves in a nonlocal Korteweg-de Vries-Burgers equation

We study travelling wave solutions of a Korteweg-de Vries-Burgers equation with a non-local diffusion term.
This model equation arises in the analysis of a shallow water flow by performing formal asymptotic expansions associated to the triple-deck regularisation (which is an extension of classical boundary layer theory). The resulting non-local operator is of fractional differential type with order between 1 and 2. Travelling wave solutions are typically analysed in relation to shock formation in the full shallow water problem.
We show rigorously the existence of these waves in the case of a quadratic nonlinearity. The travelling wave problem for the classical KdV-Burgers equation is usually analysed via a phase-plane analysis, which is not applicable here due to the presence of the non-local diffusion operator. Instead, we apply fractional calculus results available in the literature and a Lyapunov functional. In addition we discuss the monotonicity of the waves in terms of a control parameter and prove their dynamic stability in case they are monotone. We also discuss some partial results concerning the existence of travelling waves in the case of a cubic nonlinearity. This existence problem and the monotonicity of the waves in the quadratic case for a small dispersion term in relation with the diffusive one are still open problems, for this reason we have also developed numerical schemes in order to support our conjectures. We will discuss in a second part of the talk, a pseudo-spectral method that approximates the initial value problem. The basic idea is, using

an algebraic map, to transform the whole real line into a bounded interval where we can apply a Fourier expansion. Special attention is given to the correct computation of the fractional derivative in this setting. Interestingly, there is a connection of the mapping method to fractional calculus, that we will also mention.